Sem :- IV

MJC PHY :- 06

Electrodynamics & Electromagnetism (T)

Unit : 01

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Topic :-

"Maxwell's Equations" :-

Gauss's Law (Electrostatic) :-

$$\operatorname{div} E = \rho/\epsilon_0 \quad \text{or} \quad \nabla \cdot E = \frac{\rho}{\epsilon_0} \quad \text{--- (1)}$$

Gauss's Law (Magnetism) :-

$$\operatorname{div} B = 0 \quad \text{or} \quad \nabla \cdot B = 0 \quad \text{--- (2)}$$

Ampere's Law :-

$$\operatorname{Curl} B = \mu_0 J$$

$$\text{or} \quad \nabla \times B = \mu_0 \left(J + \epsilon_0 \frac{\partial E}{\partial t} \right) \quad \text{--- (3)}$$

Faraday's Law of induction

$$\operatorname{Curl} E = -\frac{\partial B}{\partial t} \quad \text{or} \quad \nabla \times E = -\frac{\partial B}{\partial t} \quad \text{--- (4)}$$

where { E = electric field
B = magnetic field

ρ = electric charge density
& J = current density

ϵ_0 = vacuum permittivity &

μ_0 = vacuum permeability.

Maxwell's eqns.

Maxwell eqns are written for fields in vacuum in the presence of electric charge of density ρ & electric current of density J. These are in macroscopic form. In the presence of simple dieletrics ϵ_0 is replaced by ϵ in eqn (1) If simple ferromagnetic materials are present, μ_0 is replaced by μ in eqn (3)

It required the genius of maxwell to show that since Ampere's law was derived for steady - state currents an additional term is required for time - varying fields for any vector, V.

$$\text{div curl } V = 0$$

from eqn (3)

$$\text{Curl } B = \mu_0 J$$

$$\text{div curl} B = \mu_0 \text{ div} J = 0 \quad (5)$$

Equation of continuity $\text{div } J + \frac{\partial \rho}{\partial t} = 0$ ——— (6)

shows that $\frac{\partial \rho}{\partial t}$ must Thus be zero, i.e the total flux of current out of any closed surface, But this certainly cannot in general be zero. because the charges can be move of from one place to another.

Maxwell purpose that eqn (3) was not correct for moving charges & suggested that similar to the electric field due to changing magnetic field. [Fraday's law of induction] there would be a magnetic field due to the changing electric field i.e.

$$\text{Curl } B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \quad \text{——— (7)}$$

The modified Ampere's law becomes

$$\text{Curl } B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

——— (8)

$$\operatorname{div}(\operatorname{Curl} B) = \operatorname{div}(\mu_0 J) + \operatorname{div}\left(\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$= \mu_0 \operatorname{div} J + \mu_0 \operatorname{div}\left(\epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$= \mu_0 \operatorname{div}(J) + \mu_0 \operatorname{div}\left(\frac{\partial D}{\partial t}\right)$$

$$= \mu_0 \left[\operatorname{div}(J) + \frac{\partial \rho}{\partial t}\right] \quad (9)$$

as $\operatorname{div}(D) = \rho$

As div (Curl B) is zero hence right hand is zero which is also zero by virtue of continuity equation Maxwell called the added term in eqn (8) the displacement current density